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College Entrance Examination Board

Advanced Placement Examination

MATHEMATICS: CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED O THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

1. Given the function f defined by $f(x) = \ln(x^2 - 9)$.
   (a) Describe the symmetry of the graph of f.
   (b) Find the domain of f.
   (c) Find all values of x such that $f(x) = 0$.
   (d) Write a formula for $f^{-1}(x)$, the inverse function of f, for $x > 3$.

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2. A particle moves along the X-axis in such a way that its position at time t for $t \geq 0$ is given by $x = \frac{1}{3}t^3 - 3t^2 + 8$
   (a) Show that at time $t = 0$ the particle is moving to the right.
   (b) Find all values of t for which the particle is moving to the left.
   (c) What is the position of the particle at time $t = 3$?
   (d) When $t = 3$, what is the total distance the particle has traveled?

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3. Given the function f defined for all real numbers x by $f(x) = 2|x - 1| x^2$.
   (a) What is the range of the function?
   (b) For what values of x is the function continuous?
   (c) For what values of x is the derivative of f(x) continuous?
   (d) Determine the value of $I = \int_0^1 f(x)\,dx$.

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4. Given the function defined by $y = x + \sin x$ for all x such that $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.
   (a) Find the coordinates of all maximum and minimum points on the given interval. Justify your answers.
   (b) Find the coordinates of all points of inflection on the given interval. Justify your answers.
   (c) On the axes provided, sketch the graph of the function.

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5. The line $x = c$ where $c > 0$ intersects the cubic $y = 2x^3 + 3x^2 - 9$ at point P and the parabola $y = 4x^2 + 4x + 5$ at point Q.
   (a) If a line tangent to the cubic at point P is parallel to the line tangent to the parabola at point Q, find the value of c where $c > 0$.
   (b) Write the equations of the two tangent lines described in (a).